University of Nottingham
Department of Mechanical, Materials and Manufacturing Engineering
MMME3086 - Computer Modelling Techniques

Exercise Sheet (FE)-1
(Pin-Jointed Elements)

1 . 1D pin-jointed problem

A 1D pin-jointed structure made up of two bar.is shown below. The top point (node 1) is fixed to a rigid surface, while a uniaxial force W is applied at the other end (node 3).

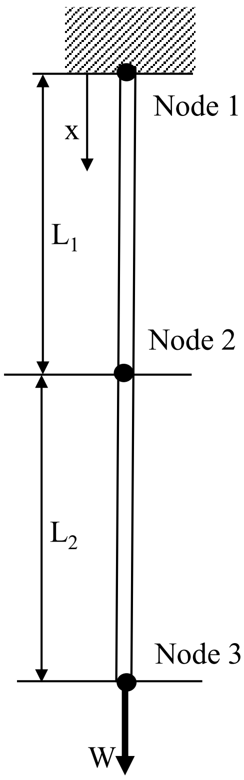
The lengths are L_1 = 2 m, L_2 = 1.5 m, and the cross-sectional areas are A_1 = 200 mm^2 and A_2 = 350 mm^2. The two bars are made of different materials with E_1= 220 GPa and E_2 = 90 GPa. The applied force is W = 50 kN.

(a) Using a FE formulation, write down the element stiffness matrix for each member.

(b) Assemble the overall stiffness matrix for the whole structure.

(c) Calculate the displacements at nodes 2 and 3.

[*Answer : (c) 2.27 mm, 4.65 mm*]



2. 2D Structural analysis problem (2 bars)

A two-bar frictionless pin-jointed structure is shown below. Points 1 and 2 are fixed to a rigid surface, and a weight of 20 kN is attached to point 3.

Both bars have a cross-sectional area of 250 mm^2 and Young's modulus of 200 GPa.

(a) Write the element stiffness matrix for each bar.

(b) Write the overall assembly stiffness matrix for the whole structure.

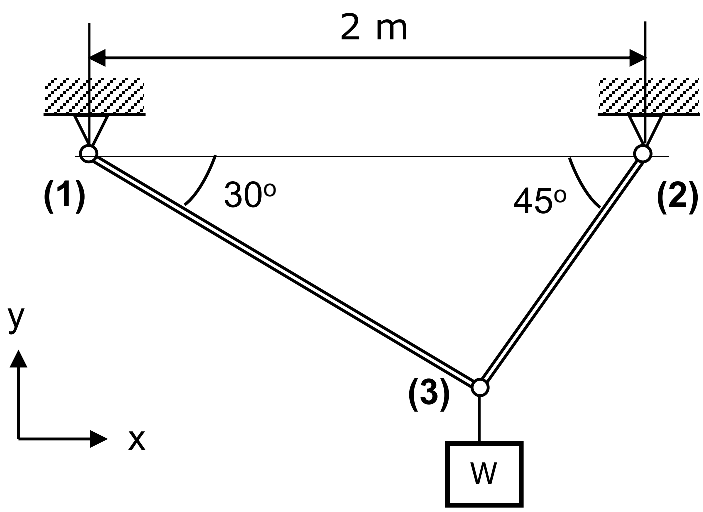
(c) Calculate the horizontal and vertical components of the displacement at point 3.

For a pin-jointed element, the element stiffness matrix $[k_e]$ is:

$$[k_e] = \left(\frac{A_e E}{L_e}\right) \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

where A_e is the cross-sectional area of the element, E is Young's modulus, and L_e is the length of the element. The angle θ is defined as the angle of inclination of the element measured anti-clockwise from the horizontal axis.

[*Answer: 0.122 mm, -0.65 mm*]



3. 2D Structural analysis problem 1 (3 bars)

A frictionless pin-jointed structure consisting of three members is shown below. Points 1, 3 and 4 are fixed to a rigid surface, and a point force of 20 kN is applied to point 2 at an angle of 30°.

The members have either a cross-sectional area of A_1 (250 mm²) or A_2 (400 mm²). Young's modulus for all members is 200 GN/m².

(a) Write down the element stiffness matrix for each member,

(b) Assemble the overall stiffness matrix for the whole structure.

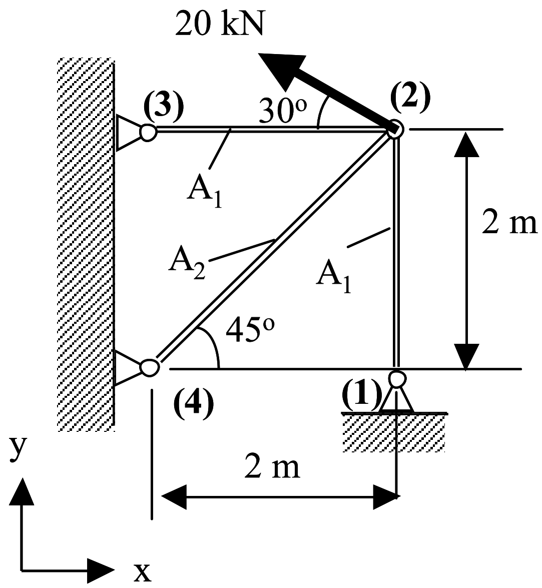
(c) Calculate the horizontal and vertical components of the displacement at point 2

For a pin-jointed element, the element stiffness matrix $[k_e]$ is:

$$[k_e] = \left(\frac{A_e E}{L_e}\right) \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

where A_e is the cross-sectional area of the element, E is Young's modulus, and L_e is the length of the element. The angle θ is defined as the angle of inclination of the element measured anti-clockwise from the horizontal axis.

[*Answer: (c) -0.615 mm, 0.478 mm*]



4. 2D Structural analysis problem 2 (3 bars)

A three-bar frictionless pin-jointed structure is shown below. Pin-joints 1, 3 and 4 are fixed to rigid surfaces. A point force of 50 kN is applied to pin-joint 2 at an angle of 30° as shown. All members have a cross-sectional area of either A_1 (150 mm^2) or A_2 (200 mm^2). Young's modulus for all members is 200 GPa.

(a) Write down the element stiffness matrix for each individual member.

(b) Assemble the overall stiffness matrix for the whole structure.

(c) Calculate the x and y components of the displacement of point 2.

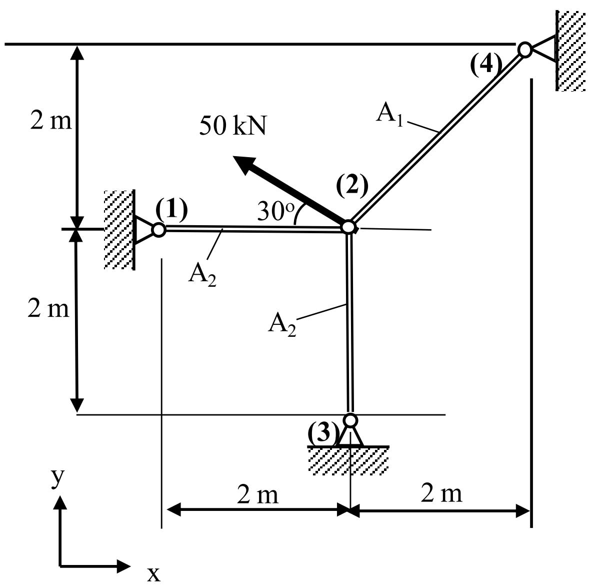
(d) Using the calculated displacements, calculate the stress in the element joining points 2 and 4. State whether the stress is compressive or tensile.

For a pin-jointed element, the element stiffness matrix $[k_e]$ can be written as follows:

$$[k_e] = \left(\frac{A_e E}{L_e}\right) \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

where A_e is the cross-sectional area of the element, E is Young's modulus, and L_e is the length of the element. The angle θ is defined as the angle of inclination of the element measured anti-clockwise from the horizontal axis.

[*Answer: (c) -2.006 mm, 1.408 mm (d) 29.89 MPa, tensile*]



5. Torsional Element

The figure below shows a two-node 'torsional' element which is a uniform circular shaft of length L under a torque T, with an angle of rotation, θ, about the cylinder axis.

The twist angle, θ, is used as the independent variable (instead of the displacement, u, in a conventional FE formulation). The applied torque, T, is equivalent to the external force, F, in a conventional FE formulation.

Over the element, the twist angle is assumed to be a linear function of x, as follows:

$$\theta = C_1\, x + C_2$$

where C_1 and C_2 are constants.

(a) Using a FE approach incorporating the principle of minimum total potential energy, derive a relationship between the applied torque and the angular displacement in the following form:

$$[T]=[K_T]\,[\theta]$$

where $[T]$ is the element torque (load) vector, $[\theta]$ is the element rotation vector and $[K_T]$ is the torsional stiffness matrix.

For a circular bar in torsion, the total potential energy is given by:

$$T.P.E. = \int_0^L \left[\frac{GJ}{2} \left(\frac{d\theta}{dx} \right)^2 \right] dx - T_1\,\theta_1 - T_2\,\theta_2$$

(b) Check your answer by using the classical torsion equation:

$$\frac{T}{J} = \frac{G\theta}{L}$$

where J is the polar second moment of area and G is the shear modulus.

Answer: $\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \left(\frac{GJ}{L} \right) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$